International Journal of Theoretical Physics, Vol. 38, No. 10, 1999

One-Loop Quantum Corrections to Thermodynamics of Black Holes with Global Monopoles

Huang Yibin[1] and Jiliang Jing[1,2]

Received April 9, 1999

Quantum corrections are studied for a black hole with a global monopole charge in a 2D model obtained by spherisymmetric reduction of the 4D action. The backreaction of the Hawking radiation on the geometry is studied perturbatively for conformal matter. It is shown that the metric and the position of horizon change by an amount of order $\hbar$. Within the off-shell approach the one-loop thermodynamic quantities, energy, and entropy are found. They are shown to contain two parts, one due to the hole itself and one to the hot gas surrounding it. The deviation of the quantum-corrected entropy from the classical one is given.

1. INTRODUCTION

Hawking [1] gave the Bekenstein–Hawking (BH) entropy of a black hole: one fourth of the area of the horizon. In processes involving a black hole, its entropy plays a role on an equal footing with the entropy of conventional matter.

The role of quantum effects in black hole physics is twofold. The backreaction of the Hawking radiation field will deform the classical black hole geometry, which was not considered by Hawking. On the other hand, the quantum correction leads to modification of the gravitational effective action, and hence of the energy and entropy.

The formal derivation of the thermal properties of a black hole is performed in the framework of the Euclidean approach initiated by Gibbons and Hawking [2, 3] which entails closing the Euclidean time coordinate with a

[1] Physics Department and Institute of Physics, Hunan Normal University, Changsha Hunan 410081, China.
[2] Department of Astronomy and Applied Physics, University of Science and Technology of China, Hefei, Anhui 230026, China.

0020-7748/99/1000-254 9$16.00/0

period $2\pi\beta = T^{-1}$. In the black hole case for arbitrary β this procedure leads to an effective Euclidean manifold which has a conical singularity at the horizon that vanishes for a fixed value $\beta = \beta_H$. Thermodynamic quantities are calculated by differentiating the corresponding free energy F with respect to β and then setting $\beta = \beta_H$ [4–6].

The well-known two-dimensional (2D) Polyakov–Liouville(PL) action [7] for conformal matter incorporates both the Hawking radiation [8] and its backreaction on the geometry (see, e.g. ref. 9). Therefore, its inclusion in the action on an equal footing with the classical counterpart gives the complete semiclassical description to the black hole. Besides the famous "string-inspired" 2D CGHS model [10] and improved RST model [11], the spherisymmetric reduction of 4D Einstein theory to an effective 2D theory of dilaton type [12–14] allows one to find the corresponding quantum deformation of the classical Schwarzschild configuration [14].

A monopole is a type of defect arising from phase transitions in the early universe. Barriola and Vilenkin (BV) [15] found a monopole solution resulting from the spontaneous breaking of symmetry. The spacetime of a global monopole asymptotes to a locally flat spacetime with a deficit solid angle of $8\pi G\eta$ [15]. Putting global charge onto the Schwarzschild black hole will amount to breaking the vacuum and asymptotic flatness. Classical thermodynamics of black holes with global monopoles is studied in ref. 16.

In this paper we use the 2D model to study the one-loop quantum effects in the thermodynamics of a black hole with global monopole charge, including the backreaction effects [6, 17, 18]. Reducing the 4D classical action spherically to an effective 2D one, we obtain the BV monopole solution (which can describe black holes with internal global monopoles), and reformulate the thermodynamics of the classical black hole in the framework of the conical singularity method. We use the PL action for the conformal quantized field and calculate the deformation of the geometry and the quantum-corrected energy and entropy of the black hole.

2. SPHERICALLY SYMMETRIC REDUCTION OF 4D THEORY

The 4D Einstein gravity coupled with a monopole is described by the action with the boundary term [2]

$$W_{\rm cl} = -\frac{1}{16\pi G}\int_{M^4} R^{(4)} + \int_{M^4} L - \frac{1}{8\pi G}\int_{\partial M^4} K^{(4)} \tag{2.1}$$

where $R^{(4)}$ is the 4D scalar curvature. $K^{(4)} = n^{\mu}_{;\mu}$ is the trace of the extrinsic curvature of the boundary ∂M^4 and n^{μ} is the outward unit vector normal to ∂M^4. The simplest Lagrangian that gives global monopoles is [15]

$$L(\psi^i) = \frac{1}{2}\psi^i_{;\mu}\psi^{i;\mu} + \frac{\lambda}{4}(\psi^i\psi^i - \eta^2)^2 \tag{2.2}$$

where ψ^i ($i = 1, 2, 3$) is the isoscalar triplet of real fields, and η is the energy scale of the symmetry breaking. This model has a global O(3) symmetry which is spontaneously broken to a global U(1) symmetry by a choice of vacuum $|\psi^i| = \eta$. The action (2.1) and the one-loop effective action are divergent when the boundary ∂M goes to infinity, which requires some subtraction procedure [19]:

$$W_{\text{sub}} = W[g_{\mu\nu}] - W[g^0_{\mu\nu}] \tag{2.3}$$

where $g^0_{\mu\nu}$ is a specially chosen background metric which is nonflat in general.

We consider the spherically symmetric metric

$$ds^2 = \gamma_{\alpha\beta}(z)\, dz^\alpha\, dz^\beta + r^2(z)(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{2.4}$$

where $\alpha, \beta, \ldots = 0, 1$, $\gamma_{\alpha\beta}(z)$ is the 2D metric on the effective 2D space M^2 covered by coordinates $z^\alpha = (\tau, x)$, and $r^2(z)$ is the scalar field. For the boundary, $\partial M^4 = \partial M^2 \times S^2$ and $n^\mu = (n^\alpha, 0, 0)$. Accordingly, the spherically symmetric field configuration describing a monopole may be written as

$$\psi^i = \eta f(r)\hat{x}^i \tag{2.5}$$

where $\hat{x}^i$ is the unit radial vector in the internal space.

After complicated calculations, we find that the action (2.2) reduces to the effective 2D theory

$$W_{\text{cl}} = \frac{1}{4G}\int_{M^2}\left[-r^2R - 2(\nabla r)^2 - 2 + \varepsilon r^2(\nabla f)^2 + 2\varepsilon f^2 + \frac{1}{2}p\varepsilon r^2(f^2 - 1)^2\right] - \frac{1}{2G}\int_{\partial M^2} kr^2 \tag{2.6}$$

(with $\varepsilon = 8\pi G\eta^2$, $p = \lambda\eta^2$) where all the geometrical objects R, ∇, k are defined with respect to the 2D space M^2 and the field $r^2(z)$ plays the role of the dilaton field. Variations of the action (2.6) with respect to the dilaton r^2, f, and metric $\gamma_{\alpha\beta}$ give, respectively,

$$rR - 2\Box r - \varepsilon r(\nabla f)^2 - \frac{1}{2}p\varepsilon r(f^2 - 1)^2 = 0 \tag{2.7}$$

$$-2r\gamma^{\alpha\beta}r_{,\alpha}f_{,\beta} - r^2\Box f + 2f + pr^2 f(f^2 - 1) = 0 \tag{2.8}$$

$$G_{\alpha\beta} = -rr_{;\alpha\beta} - \frac{1}{2}\varepsilon r^2 f_{,\alpha}f_{,\beta} + \frac{1}{2}\gamma_{\alpha\beta}[\Box r^2 - (\nabla r)^2 - 1$$

$$+\frac{1}{2}\varepsilon r^2(\nabla f)^2+\varepsilon f^2+\frac{1}{4}p\varepsilon r^2(f^2-1)^2]=0 \tag{2.9}$$

If the metric is of the form

$$ds^2=B(r)\,d\tau^2+\frac{1}{A(r)}\,dr^2 \tag{2.10}$$

(2.7) and (2.8) become

$$-r\sqrt{\frac{A}{B}}\left(\sqrt{\frac{A}{B}}B'\right)'-2\sqrt{\frac{A}{B}}(\sqrt{AB})'-\varepsilon Arf'^2-\frac{1}{2}p\varepsilon r(f^2-1)^2=0 \tag{2.11}$$

$$-2Arf'-r^2\sqrt{\frac{A}{B}}(\sqrt{AB}f')'+2f+pr^2f(f^2-1)=0 \tag{2.12}$$

and the $(\tau\tau)$ and (rr) components of (2.9) are

$$-rA'+1-A=\varepsilon f^2+\frac{1}{2}\varepsilon Ar^2f'^2+\frac{1}{4}p\varepsilon r^2(f^2-1)^2 \tag{2.13}$$

$$\frac{rAB'}{B}-1+A=-\varepsilon f^2+\frac{1}{2}\varepsilon Ar^2f'^2-\frac{1}{4}p\varepsilon r^2(f^2-1)^2 \tag{2.14}$$

from which we have

$$\frac{B'}{B}-\frac{A'}{A}=\left[\ln\left(\frac{B}{A}\right)\right]'=\varepsilon rf'^2 \tag{2.15}$$

Note that $f(r)\sim 1-(1/pr^2)$ as $r\to\infty$ because of (2.11). So $f'\approx 0$ and we obtain from (2.15) that $B/A=C$. Redefining r, we can choose $C=1$. It follows from (2.11) or (2.12) that

$$B=A=1-\varepsilon-\frac{2GM}{r} \tag{2.16}$$

where M is a constant of integration, the ADM mass of the monopole, which is negative. On the other hand, the metric (2.16) with a large value of M describes a black hole of mass M carrying a global monopole charge [15], which we are going to study. Such a black hole can be formed if a global monopole is swallowed by an ordinary black hole.

3. TREE-LEVEL BLACK HOLE THERMODYNAMICS

The Euclidean action (2.6) is the starting point for the formulation of the classical thermodynamic properties of the black hole. The standard procedure for describing the thermodynamic properties of a field system is to go to the Euclidean space by a Wick rotation $t = i\tau$ and to close the τ direction with period $2\pi\beta = T^{-1}$, where T is the temperature of the system. The system is assumed to be contained in a box of size L. In principle the field configuration does not necessarily satisfy any field equations. The latter arise as a requirement of extremality of the free energy functional under appropriately defined boundary conditions.

Analogously the thermodynamics of black holes can be formulated off-shell. Consider the Euclidean static metric of the general type

$$ds^2 = g(x)\, d\tau^2 + \frac{e^{-2\lambda(x)}}{g(x)}\, dx^2 \tag{3.1}$$

where $0 \le \tau \le 2\pi\overline{\beta}$, $x_+ \le x \le L$, and $x = x_+$ is the location of the horizon. A nonextremal black hole requires that at the horizon $x = x_+$ the function $g(x)$ has a simple zero. Changing the variable x to ρ so that

$$d\rho = \frac{dx}{\sqrt{g(x)e^{\lambda(x)}}} \tag{3.2}$$

we see that, in the vicinity of the horizon, the metric becomes

$$ds^2 = d\rho^2 + \alpha^2\rho^2\, d\phi^2 \tag{3.3}$$

where $\alpha = \overline{\beta}/\beta_H$, $\beta_H = 2/g'_x e^{\lambda}\big|_{x=x_+}$, and $\phi = \tau/\overline{\beta}$. Hence the metric (3.1) describes the Euclidean space with conical singularity at the horizon with angle deficit $\delta = (1 - \alpha)2\pi$, which vanishes when $\alpha = 1$. This implies that the scalar curvature is of the form [20]

$$R = 4\pi(1 - \alpha)\delta_\Sigma + \overline{R} \tag{3.4}$$

where $\int_{M_\alpha} \delta_\Sigma = 1$, $\int_{M_\alpha} \delta_\Sigma f = f_\Sigma$ (f_Σ is the value of f on horizon Σ), and $\overline{R}$ is the regular part.

The system is specified by fixing (1) the temperature T and values of f_B and r_B on the external boundary and (2) the black hole topology (i.e., nonextremal case), while all the functions (g, λ, r, f) and the values of $r_+ = r(x_+)$, $g'(x_+)$ (or $\beta_H \equiv 2e^{-\lambda}/g'_x\big|_{x=x_+}$) are variable. Thus, our approach includes both the regular and conically singular ($\alpha \neq 1$) metric (in other words the calculations are done off-shell), while in ref. 21 only regular metrics are considered. For a metric with an arbitrary α the classical action (2.6) due to (3.4) takes the form

$$W_{\rm cl} = \frac{1}{4G}\int_{\overline{M}}[-r^2R - 2(\nabla r)^2 - 2 + \varepsilon r^2(\nabla f)^2 + 2\varepsilon f^2$$

$$+ \frac{1}{2}p\varepsilon r^2(f^2 - 1)^2] - \frac{1}{2G}\int_{\partial\overline{M}} r^2 k^{(2)} - \frac{\pi r_+^2}{G}(1 - \alpha) \qquad (3.5)$$

For the static metric (3.1), the action (3.5) becomes

$$W_{\rm cl} = \frac{\pi\overline{\beta}}{G}\int_{x+}^{L}[-g_x' e^{\lambda} rr' - ge^{\lambda}r'^2 - e^{-\lambda} + \frac{1}{2}\varepsilon r^2 ge^{\lambda}f'^2$$

$$+ e^{-\lambda}\varepsilon f^2 + \frac{1}{4}e^{-\lambda}p\varepsilon r^2(f^2 - 1)^2]\, dx - \frac{\pi r_+^2}{G} \qquad (3.6)$$

One can define the free energy F, entropy S, and energy E associated with $W_{\rm cl}$ as

$$F = (2\pi\beta)^{-1}W_{\rm cl}, \qquad S = (\beta\partial_\beta - 1)W_{\rm cl}, \qquad E = \frac{1}{2\pi}\partial_\beta W_{\rm cl} \qquad (3.7)$$

where $2\pi\beta = T^{-1}$ and $\beta = \overline{\beta}g_B^{1/2}$. Hence we have for the energy E

$$E = \frac{1}{2Gg_B^{1/2}}\int_{x+}^{L}\left[-\frac{1}{2}(r^2)'e^{\lambda}g' - ge^{\lambda}(r_x')^2 - e^{-\lambda}\right.$$

$$\left. + \frac{1}{2}\varepsilon r^2 ge^{\lambda}f'^2 + e^{-\lambda}\varepsilon f^2 + \frac{1}{4}e^{-\lambda}p\varepsilon r^2(f^2 - 1)^2\right] dx \qquad (3.8)$$

and for the entropy

$$S_{\rm BH} = \frac{\pi r_+^2}{G} \qquad (3.9)$$

which takes the standard BH form. Now we fix the temperature T and consider the extremum of the free energy $F = E - TS$ or equivalently that of the action $W_{\rm cl}$. Such an equilibrium configuration automatically satisfies the second law of black hole thermodynamics:

$$\delta E = T\delta S \qquad (3.10)$$

The total variation of the action $W_{\rm cl}$ is $\delta W_{\rm cl} = \delta_r W_{\rm cl} + \delta_g W_{\rm cl} + \delta_\lambda W_{\rm cl} + \delta_f W_{\rm cl}$. For partial variations we have

$$\delta_r W_{\rm cl} = \frac{2\pi r(x_+)}{G}(1-\alpha)\,\delta r(x_+) + \frac{\pi\bar\beta}{G}\int_{x+}^{L}\Bigg[-g_x' e^\lambda r' + (g_x' e^\lambda r)' + 2(ge^\lambda r')' + \varepsilon rge^\lambda f'^2 + \frac{1}{2}e^{-\lambda}p\varepsilon r(f^2-1)^2\Bigg]\,\delta r\,dx \tag{3.11}$$

$$\delta_f W_{\rm cl} = \frac{\pi\bar\beta}{G}\int_{x+}^{L}[-(\varepsilon r^2 ge^\lambda f')' + 2e^{-\lambda}\varepsilon f + e^{-\lambda}p\varepsilon r^2 f(f^2-1)]\,\delta f\,dx \tag{3.12}$$

$$\delta_g W_{\rm cl} = \frac{\pi\bar\beta}{G}\int_{x+}^{L}\Bigg[(e^\lambda rr')' - e^\lambda r'^2 + \frac{1}{2}\varepsilon r^2 e^\lambda f'^2]\,\delta g\,dx \tag{3.13}$$

$$\delta_\lambda W_{\rm cl} = \frac{\pi\bar\beta}{G}\int_{x+}^{L}[-g_x' e^\lambda rr' - ge^\lambda r'^2 + e^{-\lambda} + \frac{1}{2}\varepsilon r^2 e^\lambda gf'^2 - e^{-\lambda}\varepsilon f^2 - \frac{1}{4}e^{-\lambda}p\varepsilon r^2(f^2-1)^2\Bigg]\,\delta\lambda\,dx \tag{3.14}$$

These lead to four equations of motion which of course coincide with (2.7)–(2.9) written for the metric(3.1). Moreover, the requirement $\delta_r W_{\rm cl} = 0$ gives the condition $\alpha = 1$. It means that the equilibrium state is reached on a regular manifold without conical singularity (Gibbon–Hawking instanton).

Using the last two equations of motion resulting from (3.13) and (3.14), the energy functional E of (3.8) reduces to the surface terms only:

$$E = E_{\rm surf} = -\sqrt{g}e^\lambda rr'\Big|_{x=L} \tag{3.15}$$

Equivalently, we obtain a coordinate-invariant expression for the energy (3.15):

$$E = -\frac{1}{2\pi\beta G}\int_{\partial M} rn^\alpha r_{,\alpha} \tag{3.16}$$

The subtraction procedure described in Section 2 leads to the result

$$\begin{aligned} E &= E[g] - E[g_0] \\ &= \frac{1}{G}\left(\frac{1}{2\pi\beta_0}\int_{\partial M} rn_0^\alpha r_{,\alpha} - \frac{1}{2\pi\beta}\int_{\partial M} rn^\alpha r_{,\alpha}\right) \\ &= \frac{1}{G}[r(g_0^{1/2} - g^{1/2})]_{r=L} \end{aligned} \tag{3.17}$$

where we chose $r_0 = r$ for the reference metric. Note that the natural condition to be imposed on the background is that in the limit $L \to \infty$ the background temperature $T = (2\pi\,\beta_0)^{-1}$ coincides with the black hole temperature measured at infinity. This is satisfied if $g_0 = \lim_{L\to\infty} g(L)$. For the monopole case (2.16), we have $g_0 = 1 - \varepsilon$. Hence for the energy we find in the limit $L \to \infty$ that

$$E = M/\sqrt{1-\varepsilon} \tag{3.18}$$

4. QUANTUM-CORRECTED BLACK HOLE GEOMETRY

In the semiclassical approximation the one-loop quantum effects are taken into account by adding to the classical action the quantum counterpart obtained by integrating out the matter fields:

$$W = W_{\rm cl} + \Gamma \tag{4.1}$$

We take the classical part $W_{\rm cl}$ to have the form (2.6), while the one-loop contribution Γ is the nonlocal PL action [7] $\Gamma_{\rm PL}[g] = (1/96\pi) \int R\Box^{-1}R$ for a 2D quantum conformal massless scalar field. Adding to it the boundary terms to specify the state of the quantum field, and considering the conical singularity on the horizon, the PL action becomes [18]

$$\begin{aligned}\Gamma[M_\alpha] = &\frac{1}{48\pi}\int_{M_\alpha}\left[\frac{1}{2}(\nabla\psi)^2 + \psi\bar{R}\right] + \frac{1}{24}\frac{(1-\alpha)^2}{\alpha}\psi_h \\ &+ \frac{1}{24\pi}\int_{\partial M_\alpha}\kappa\psi + \frac{1}{16\pi}\int_{\partial M_\alpha} n^\mu\psi_{,\mu} + \Gamma_0\end{aligned} \tag{4.2}$$

where $\psi(x)$ is the solution of the equation $\Box\psi = R \equiv 4\pi(1-\alpha)\,\delta_\Sigma + \bar{R}$.

We first study the corrections to the classical geometry of the black hole induced by quantum corrections to the action (4.1). Variation of (6.1) with respect to the metric gives the equations

$$G_{\alpha\beta} = -T_{\alpha\beta} \tag{4.3}$$

$$T_{\alpha\beta} = \frac{\kappa}{2}\left\{2\psi_{;\alpha\beta} - \psi_{,\alpha}\psi_{,\beta} - \gamma_{\alpha\beta}\left[2R - \frac{1}{2}(\nabla\psi)^2\right]\right\} \tag{4.4}$$

where $G_{\alpha\beta}$ is given by (2.9) and $\kappa = G/24\pi$, while variations with respect to the dilaton field $r^2(x)$ and $f(x)$ give the same equations as in the classical case [see (2.7) and (2.8)].

Note that, comparing the trace of (4.3),

$$\Box r^2 - 2 + 2\varepsilon f^2 + \frac{1}{2} p\varepsilon r^2 (f^2 - 1)^2 = 2\kappa R \tag{4.5}$$

with (2.7) and (2.8) we obtain for the curvature

$$R = \frac{2 - 2(\nabla r)^2 + \varepsilon r^2 (\nabla f)^2 - 2\varepsilon f^2}{r^2 - 2\kappa} \tag{4.6}$$

which implies that the spacetime singularity now is placed at finite radius (value of the dilaton) $r^2 = r_{\rm cr}^2 \equiv 2\kappa$. This typically happens in 2D models of gravity, as has been previously observed [12, 14, 22]. Hence we assume that the horizon lies at $r_+ \gg r_{\rm cr}$. Then, in the region $r \geq r_+$ we may solve (4.3), (2.7), and (2.8) perturbatively (with respect to $r_{\rm cl}/r_+$) considering $T_{\alpha\beta}$ in the right-hand side of (4.3) as a small perturbation and take them on the classical background. This gives the correction to the black hole geometry to first order in the Planck constant $\hbar$.

As earlier we consider a static solution:

$$ds^2 = C(r)\, d\tau^2 + \frac{dr^2}{D(r)} \tag{4.7}$$

Then (4.3), (2.7), and (2.8) become

$$-rD' - D + 1 - \varepsilon f^2 - \frac{1}{2}\varepsilon D r^2 f'^2 - \frac{1}{4} p\varepsilon r^2 (f^2 - 1)^2 = 2T_\tau^\tau \tag{4.8}$$

$$\frac{rDC'}{C} + D - 1 + \varepsilon f^2 - \frac{1}{2}\varepsilon D r^2 f'^2 + \frac{1}{4} p\varepsilon r^2 (f^2 - 1)^2 = -2T_r^r \tag{4.9}$$

$$-r\sqrt{\frac{D}{C}}\left(\sqrt{\frac{D}{C}}C'\right)' - 2\sqrt{\frac{D}{C}}(\sqrt{CD})' - \varepsilon D r f'^2 - \frac{1}{2} p\varepsilon r (f^2 - 1)^2 = 0 \tag{4.10}$$

$$-2Drf' - r^2\sqrt{\frac{D}{C}}(\sqrt{CD}f')' + 2f + pr^2 f(f^2 - 1) = 0 \tag{4.11}$$

It follows from (4.8) and (4.9) that

$$\left(\ln\frac{C}{D}\right)' - \varepsilon r f'^2 = \frac{2(T_\tau^\tau - T_r^r)}{Dr} \tag{4.12}$$

Setting $D = 1 - \varepsilon - 2M(r)G/r$, we have from (4.8)

$$2GM' - \varepsilon(f^2 - 1) - \frac{1}{4} p\varepsilon r^2 (f^2 - 1)^2 - \frac{1}{2}\varepsilon D r^2 f'^2 = 2T_\tau^\tau \tag{4.13}$$

At the classical level, the stress tensor $T_{\alpha\beta}$ of (4.4) reads

$$T_\tau^\tau = \kappa\left[g'' - \frac{1}{4g}\left(g'^2 - (g'_+)^2\right)\right]$$
$$T_r^r = \frac{\kappa}{4g}\left(g'^2 - (g'_+)^2\right) \tag{4.14}$$

where $g = 1 - \varepsilon - 2MG/r = (1 - \varepsilon)(1 - r_+/r)$ and $r_+ = 2MG/(1 - \varepsilon)$. To second order in r^{-1}, we have

$$T_\tau^\tau \approx \frac{\kappa(1 - \varepsilon)}{4r_+^2}\left(1 + \frac{r_+}{r} + \frac{r_+^2}{r^2}\right)$$
$$T_r^r \approx -\frac{\kappa(1 - \varepsilon)}{4r_+^2}\left(1 + \frac{r_+}{r} + \frac{r_+^2}{r^2}\right) \tag{4.15}$$

Now we assume that f still behaves like $1 - (1/pr^2)$ asymptotically and $f' \approx 0$, which can be justified at the end. Then, from (4.13) we have, to first order in r^{-1},

$$GM(r) \approx \int T_\tau^\tau\, dr \approx GM + \kappa m(r) \tag{4.16}$$

where

$$m(r) = \frac{1 - \varepsilon}{4}\left(\frac{r}{r_+^2} + \frac{1}{r_+}\ln\frac{r}{l}\right) \tag{4.17}$$

We have introduced a distance l in order to have dimensionless quantities under the logarithms. So D becomes, to r^{-1},

$$D = 1 - \varepsilon - \frac{2[MG + \kappa m(r)]}{r}$$
$$= (1 - \varepsilon)\left(1 - \frac{\kappa}{2r_+^2}\right) - \frac{\kappa(1 - \varepsilon)}{2r_+ r}\ln\frac{\mathrm{r}}{1} - \frac{2MG}{r} \tag{4.18}$$

Setting

$$C(r) = D(r)e^{\Phi(r)} \tag{4.19}$$

we have from (4.12) that

$$\Phi' \approx \frac{\kappa}{r_+^2}\left(\frac{1}{r} + \frac{2r_+}{r^2} - \frac{3r_+^2}{r^3}\right)$$

Imposing on $\Phi(r)$ the condition $\Phi(L) = 0$, we get

$$\Phi(r) = \kappa[F(L) - F(r)] \tag{4.20}$$

where

$$F(r) = \frac{2}{r_+ r} + \frac{3}{2r^2} - \frac{1}{r_+^2} \ln \frac{r}{l} \tag{4.21}$$

Setting $D(r) = 0$, we find that the deformed horizon is now located at

$$\bar{r}_+ = \frac{2(MG + \kappa m(\bar{r}_+))}{1 - \varepsilon} \approx \frac{2(MG + \kappa m(r_+))}{1 - \varepsilon} = r_+ + \frac{2\kappa}{1 - \varepsilon} m(r_+) \tag{4.22}$$

So, due to quantum effects, r_+ is changed by

$$\frac{2\kappa}{1 - \varepsilon} m(r_+) = \frac{\kappa}{2r_+}\left(1 + \ln \frac{r_+}{l}\right) \tag{4.23}$$

5. QUANTUM CORRECTIONS TO BLACK HOLE THERMODYNAMICS

Our approach to the one-loop thermodynamics described by the action W (4.1) is essentially the same as in the tree-level approximation considered in Section 3. We fix r_B, f_B, and $T = (2\pi\beta)^{-1}$ on the boundary $x = L$ and the black hole topology, and define the off-shell entropy and energy by the relations

$$S = (\beta\partial_\beta - 1)W, \qquad E = \frac{1}{2\pi}\,\partial_\beta W \tag{5.1}$$

Taking the Euclidean static metric in the form (3.1), we find that the extremum of the functional $W[g(x), r(x), \lambda(x), f(x)]$ as in the classical case is attained on the regular manifold, where now $r(x_+) = \bar{r}_+$ [see (3.11)].

Calculating the off-shell quantities (5.1), it is convenient to write the metric in the Schwarzschild-like form:

$$ds^2 = g(x)\, d\tau^2 + g^{-1}(x)\, dx^2 \tag{5.2}$$

where $0 \leq \tau \leq 2\pi\bar{\beta}$. The quantum-corrected metric found in Section 4 takes this form by means of the coordinate transformation $r \rightarrow x(r)$, $x'_r = e^{\Phi(r)}$ and identification $g(x) = De^{2\Phi}$. Then $\psi(x)$ takes the form

$$\psi_{M_\alpha}(x) = -\ln g(x) - \frac{2}{\beta}\int_x^L \frac{dx}{g(x)} - 2\ln\frac{\beta_H}{z_0} + C(\alpha, z_0) \tag{5.3}$$

where z_0 is the proper generator length of the cone conformal to M_α. The PL action Γ of (4.2) reads [18]

$$\Gamma[g] = \frac{1}{24}\int_{x_+}^{L}\left(\frac{2}{\bar{\beta}g} - \frac{\bar{\beta}}{2}\frac{g'^2}{g}\right)dx + \frac{1}{12}\left(\alpha + \frac{(1-\alpha^2)}{2\alpha}\right)\psi(x_+)$$
$$- \frac{\bar{\beta}}{8}g'(L) + \Gamma_0 \tag{5.4}$$

where Γ_0 is a conformally invariant functional. It should be noted that (5.4) is divergent at the lower limit. Taking the regularization $x_+ \rightarrow x_+ + \delta$, we have, for the divergent part of (5.4),

$$\Gamma_{\text{div}} = \ln\delta\,\frac{(1-\alpha)^2}{24\alpha^2} \tag{5.5}$$

Note that Γ_{div} is proportional to $(1-\alpha)^2$ and does not affect physical quantities calculated at the Hawking temperature ($\beta = \beta_H$).

For the equilibrium state ($\beta = \beta_H$), $E = E_{cl} + E_q$, where the classical part E_{cl} takes the form (3.8) and the quantum part reads

$$E_q = \frac{1}{2\pi}\partial_{\bar{\beta}}\Gamma\Big|_{\bar{\beta}=\beta_H} = \frac{1}{96\pi\sqrt{g(L)}}\int_{x_+}^{L}\frac{1}{g}\left(\frac{4}{\beta_H^2} - g'^2(x)\right)dx - \frac{1}{16\pi g^{1/2}(L)}g'(L) \tag{5.6}$$

which is free of divergence at the lower limit. For the quantum-corrected metric, $g'(L)$ vanishes in the limit $L \rightarrow \infty$. Therefore we neglect such a term below.

Using the equations of motion, we get

$$E = E_{\text{surf}} + \frac{T}{6} = -\frac{T}{G}\int_{\partial M} rn^{\alpha}r_{,\alpha} + \frac{T}{6} = -\frac{1}{G}rg^{1/2}\Big|_{r=L} + \frac{T}{6} \tag{5.7}$$

Note that both terms in (5.10) are defined on the external boundary $r = L$. Subtracting now the energy of the background g_0, we obtain

$$E[g] - E[g_0] = \frac{1}{G}r(g_0^{1/2} - g^{1/2})\Big|_{r=L} + \frac{1}{6}(T - T_0) \tag{5.8}$$

where $T_0 = [2\pi\beta_H^0 g_0^{1/2}(L)]^{-1}$ is the temperature of the background metric. The temperature $T = 1/2\pi\beta_H\sqrt{g(L)}$ in (5.7), though measured at the external boundary, originates from the horizon [when one integrates by parts in passage from (5.6) to (5.7)], and is a consequence of the black hole topology. In the non-black-hole case (hot space) this term is absent. Taking $T_0 = T$, we get the classical expression (3.17), but now g and g_0 are the corresponding quantum-corrected metrics.

For the quantum-corrected solution (4.18), we have

$$g(L) = D(L)\, e^{\Phi(L)} = (1 - \varepsilon)\left(1 - \frac{\kappa}{2r_+^2}\right) - \frac{2MG}{L} - \frac{\kappa(1 - \varepsilon)}{2r_+ L} \ln \frac{L}{l} \tag{5.9}$$

We see that in the limit $L \to \infty$, $g(L) \to g_0 = (1 - \kappa/2r_+^2)(1 - \epsilon)$ rather than to $(1 - \varepsilon)$. Introducing the Planck temperature $T_{\mathrm{Pl}} = (2\pi r_{\mathrm{cr}})^{-1}$, we can rewrite this as $g_0 = 1 - \varepsilon - (T_{\mathrm{H}}/T_{\mathrm{pl}})^2/(1 - \varepsilon)$. We see that the modification of the asymptotic behavior of g and of the background is essentially due to temperature effects. Indeed, if we were to take the background $g_0 = 1 - \varepsilon$ as in the classical case and apply (5.8) for the metric (5.9), we would obtain for the energy the divergent term $E_{\mathrm{th}} = \pi L T_{\mathrm{H}}^2/6(1 - \varepsilon)^{3/2}$, which is the energy of the hot gas surrounding the black hole. Hence, the system under consideration includes two objects: the black hole and hot gas. Extensive characteristics (such as energy or entropy) contain different contributions due to these two subsystems. The contribution of the hot gas depends on the size of the system L, while the contribution of the hole itself does not.

Choosing $g_0 = (1 - \kappa/2r_+^2)(1 - \varepsilon)$, we get for the energy

$$E = \frac{M}{\sqrt{1 - \varepsilon}} + \frac{\kappa M \eta}{4r_+^2 \sqrt{1 - \varepsilon}} = \frac{M}{1 - \varepsilon}\left[1 + \frac{1}{2}\left(\frac{T_{\mathrm{H}}}{T_{\mathrm{Pl}}}\right)^2 \frac{\eta}{(1 - \varepsilon)^2}\right] \tag{5.10}$$

where $\eta = 1 + 2\ln(L/l)$. We see that the contribution E_{th} of the hot gas disappears. However the logarithmically infrared divergent term is still present and will be absent when massive matter is considered.

Analogously, we have for the entropy in the equilibrium state

$$S = \frac{\pi \bar{r}_+^2}{G} + S_q \tag{5.11}$$

where

$$S_q = (\beta\partial_\beta - 1)\Gamma\big|_{\bar{\beta}=\beta_{\mathrm{H}}} = -\frac{1}{12}\,\psi(x_+)$$
$$= \frac{1}{12}\int_{x_+}^{L} \frac{dx}{g(x)}\left(\frac{2}{\beta_{\mathrm{H}}} - g'(x)\right) + \frac{1}{6}\ln\frac{\beta_{\mathrm{H}} g^{1/2}(L)}{z_0} + c(z_0) \tag{5.12}$$

Note that S_q is also free of divergence. For a metric written in the conformally flat form $g_{\mu\nu} = e^{2\sigma}\delta_{\mu\nu}$, we have $\psi(x) = -2\sigma(x)$ and the entropy (5.12) coincides with that previously obtained [20, 23].

Substituting the classical metric function $g_{\mathrm{cl}}(r) = (1 - \varepsilon)(1 - r_+/r)$ into the expression for S_{q} (which is really of the order $\hbar$), we find that

$$S_q = \frac{\pi}{3} T_H \frac{L - r_+}{1 - \varepsilon} + \frac{1}{12} \ln \frac{L - r_+}{r_+} + \frac{1}{6} \ln \frac{2r_+}{z_0 \sqrt{1 - \varepsilon}} \tag{5.13}$$

which is divergent in the limit $L \to \infty$. The first linearly divergent term is the entropy S_{th} of the 2D hot gas, and should be subtracted.

Taking $\bar{r}_+$ instead of r_+ in S_q, we derive the complete quantum entropy of the hole when $L \to \infty$:

$$S = \frac{\pi \bar{r}_+^2}{G} + \frac{1}{12} \ln \frac{L}{\bar{r}_+} + \frac{1}{6} \ln \frac{2\bar{r}_+}{z_0 \sqrt{1 - \epsilon}} \tag{5.14}$$

It can be rewritten in the form

$$S = \frac{A_+}{4G} + \frac{1}{12} \ln \frac{A_+}{\pi z_0^2 (1 - \varepsilon)} \tag{5.15}$$

where $A_+ = 4\pi \bar{r}_+^2$ is the area of the horizon and we omitted a term $\propto \ln(L/\bar{r}_+)$. This result is similar to that obtained in ref. 5 for the 4D Schwarzschild black hole.

6. CONCLUSION

The classical spacetime of black holes with global monopoles is described by the BV metric, but the backreaction of the conformal matter alters the metric and changes the position of the horizon. The equilibrium state of the black hole requires a regular manifold without conical singularity ($\alpha = 1$). So does that of the quantum-corrected hole. The classical entropy takes the BH form, but the quantum-corrected one contains an additional logarithmic term. Moreover, both energy and entropy contain two parts, one due to the hot gas and one to the hole itself, and the hot-gas part can be eliminated by choosing the reference metric appropriately.

REFERENCES

1. S. W. Hawking, *Nature* **248**, 30 (1974); *Commum. Math. Phys.* **43**, 199 (1975).
2. G. W. Gibbons and S. W. Hawking, *Phys. Rev. D* **15**, 2752 (1977).
3. S. W. Hawking, In *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979).
4. S. N. Solodukhin, *Phys. Rev. D* **51**, 609 (1995).
5. D. V. Fursaev, *Phys. Rev. D* **51**, 5352 (1995).
6. S. N. Solodukhin, *Phys. Rev. D* **53**, 824 (1996).
7. A. M. Polyakov, *Phys. Lett. B* **103**, 207 (1981).
8. V. P. Frolov and G. A. Vilkovisky, *Phys. Lett. B* **106**, 307 (1981); in *Quantum Gravity (Proceedings of the Second Moscow Quantum Gravity Seminar, Moscow, 1981)*, edited by M. A. Markov and P. C. West (Plenum Press, New York, 1983).

9. N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Spaces* (Cambridge University Press, Cambridge, England, 1982).
10. C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, *Phys. Rev. D* **45**, R1005 (1992).
11. J. G. Russo, L. Susskind, and L. Thorlacius, *Phys. Rev. D* **46**, 3444 (1992); **47**, 533 (1992).
12. D. A. Lowe, *Phys. Rev. D* **47**, 2446 (1993).
13. M. McGuigan, C. R. Nappi, and S. A. Yost, *Nucl. Phys. B* **375**, 421 (1992); O. Lechtenfeld and C. Nappi, *Phys. Lett. B* **288**, 72, (1992).
14. D. I. Kazakov and S. N. Solodukhin, *Nucl. Phys. B* **429**, 153 (1994).
15. M. Barriola and A. Vilenkin, *Phys. Rev. Lett.* **63**, 341 (1989).
16. J. L. Jing and Y. J. Wang, *Phys. Lett. A* **178**, 59 (1993); H. W. Yu, *Nucl. Phys. B* **430**, 427 (1994).
17. O. Zaslavski, *Phys. Rev. D* **53**, 4691 (1996).
18. V. P. Frolov, W. Israel, and S. N. Solodukhin, *Phys. Rev. D* **54**, 2732 (1996).
19. S. W. Hawking and G. T. Horowitz, *Class. Quantum Grav.* **13**, 1487 (1996).
20. D. V. Fursaev and S. N. Solodukhin, *Phys. Rev. D* **52**, 2133 (1995).
21. J. W. York, Jr. *Phys. Rev. D* **33**, 2092 (1986); B. F. Whiting and J. W. York, Jr., *Phys. Rev. Lett.* **61**, 1336 (1988); H. W. Braden, J. D. Brown, B. F. Whiting, and J. W. York, Jr., *Phys. Rev. D* **42**, 3376 (1990).
22. T. Banks, A Dabholkar, M. R. Douglas, and M. O'Loughlin, *Phys. Rev. D* **45**, 3607 (1992); B. Birnir, S. B. Giddings, J. A. Harvey, and A. Strominger, *Phys. Rev. D* **46**, 638 (1992); S. W. Hawking, *Phys. Rev. Lett.* **69**, 406 (1992).
23. T. M. Fiola, J. Preskill, A. Strominger, and S. P. Trivedi, *Phys. Rev. D* **50**, 3987 (1994); R. C. Myers, *Phys. Rev. D* **50**, 6412 (1994).